

A modified orbital motion limited (OML) theory

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The validity of the orbital motion limited (OML) theory is reviewed with reference to the floating potential acquired by a spherical object immersed in a plasma. A new and perhaps more realistic approach for obtaining the floating potential is introduced by including the current outward from the spherical object and the current coming from infinity. This novel approach is also valid for cases where the standard OML theory ceases to apply.

The collection of plasma particles by an ideal spherical object has applications e.g., in the operation of Langmuir probes, spacecraft and other immersed objects and, especially, for grains in dusty plasma[1–8]. Since the different objects in the mentioned phenomena behave in a similar way, we intend to use the general term ‘object’ throughout this paper.

There have been many theoretical attempts to solve the problem of the charging of objects immersed in plasmas starting from Mott-Smith and Langmuir[9], and followed by many other authors[10–20]. One needs to determine the floating potential to study the objects introduced into the plasma, that is the potential at which ion and electron currents to the object are balanced. However, there is no universally accepted theory for the quantitative description of the particle charging in plasmas. Yet, owing to its simplicity, the orbital motion limited (OML) approach is often applied for spherical objects in low density plasmas[9, 12]. This approach deals with collisionless electron and ion trajectories in the vicinity of a small object and allows us to determine the cross-sections for electron and ion collection from the conservation laws of energy and angular momentum. This seems to be quite a reasonable approach, for example in the case of dusty plasmas, where the mean free paths of the ion collisions with atoms are usually very long compared to the Debye length such that $a \ll \lambda_d \ll l_{i(e)}$ [17], where a and λ_d are the radius of object and plasma screening length (the corresponding Debye radius), respectively, and $l_{i(e)}$ denotes the mean free path of the ions (electrons). It is also assumed that the dust particle is rather isolated in the sense that other dust particles do not affect the motion of electrons and ions in its vicinity. However, the validity of the OML approach is questionable. Strictly speaking, the OML approach is valid only when there is no absorption radius, which is the absorption radius imagined as a theoretical spherical surface where the grazing orbit is the limiting orbit that an ion of a particular energy will hit the surface of object. For any distribution function containing slow ions, the absence of the absorption radius can be reformulated in a condition on the shielding potential around the body, which must decrease more slowly than $1/r^2$. Allen, Annaratone and de Angelis have shown that the OML approach is not congruent relevant to dusty plasma phenomena[21]. However, later on it was proved that with limits of a vanishing body radius, OML theory is consistent for cases relevant to dusty plasmas, and thus can be used as a useful approximation for quantitative calculations.[18].

The purpose of this letter is to raise pertinent questions on the existing OML theory and to present a new approach for obtaining the floating potential which is fundamental in understanding to occurring phenomenon.

Consider the well-known OML result that φ_s is a solution of

$$\left(\frac{T_e}{m_e}\right)^{1/2} \exp\left(-\frac{e|\varphi_s|}{K_B T_e}\right) = \left(\frac{T_i}{m_i}\right)^{1/2} \left(1 + \frac{e|\varphi_s|}{K_B T_i}\right), \quad (1)$$

where m is the particle mass, T is the temperature, and K_B is Boltzmanns constant. Here, we adopt the conventions that subscripts e and i denote properties of the plasma electrons and ions respectively. e is the magnitude of the electron charge and φ_s is the surface potential of the object. The above equation (1) seems not to be general as it is always useful to consider a specific situation, for example when $e|\varphi_s| > K_B T_e$, and on the other hand when the object is removed from a plasma i.e., $\varphi_s = 0$, it becomes invalid. Yet, it yields a good approximation and provides a good description on the surface of the object as it assumes neutrality condition ($n_{io} \simeq n_{eo}$).

Here, we suggest another derivation of the above equation for the total current since the floating condition is

$$I_i + I_e = 0, \quad (2)$$

where I_i and I_e are the ion and electron currents collected by the body. For the case when there is no object in the plasma, $I_i = 0$ and $I_e = 0$. For a spherical object with some potential around it, there can be two types of current influenced by this potential, one is I_α^- (α for the electron and ion species) which is outward from the object and I_α^+ ($\varphi_s = 0$) which comes from infinity. It is important to note that I_α^- is not due to the reflection of incoming

electrons (or ions) from the object's surface[22–24]. Fig. (1) illustrates the ion and electron currents to and from the spherical object of radius a . Effects of other currents like thermionic, secondary or photoemission, etc.[2, 25]. are ignored here and the total current is given by

$$I_{\alpha}^{total} = I_{\alpha}^{-}(\varphi_s) + I_{\alpha}^{+}(\varphi_s = 0). \quad (3)$$

If no object is present ($\varphi_s = 0$), $I_{\alpha}^{total} = I_{\alpha}^{-} + I_{\alpha}^{+} = 0$, providing $I_{\alpha}^{+} = -I_{\alpha}^{-}$. Now, using the OML approximation for the electrons, we can write

$$I_e = -4\pi r_o^2 n_{oe} e \left(\frac{m_e}{2\pi K_B T_e} \right)^{3/2} \left\{ \int_0^{\infty} v_z e^{-\frac{m_e v^2}{2\pi K_B T_e}} dv + \int_{-\infty}^0 v_z e^{-\frac{m_e v^2 + 2e|\varphi_s|}{2K_B T_e}} dv \right\}. \quad (4)$$

Integration of the above equation gives us the electron current

$$I_e = -4\pi r_o^2 n_{oe} e \left(\frac{K_B T_e}{2\pi m_e} \right)^{1/2} \left(1 - e^{-\frac{e|\varphi_s|}{K_B T_e}} \right). \quad (5)$$

Now we consider an ion approaching an object from the bulk plasma at the edge of its shielding cloud ($r = \infty$) at speed v_i and **striking** the spherical object with surface potential φ_s and speed v_i' . We assume that there are no collisions with other particles in the shielding cloud. The conservation of energy and angular momentum is expressed by the following equations

$$\frac{m_i v_i^2}{2} = \frac{m v_i'^2}{2} - e|\varphi_s|, \quad (6)$$

where from above $v_i' = v_i \left(1 + \frac{2e|\varphi_s|}{m_i v_i^2} \right)^{1/2}$, and

$$m_i v_i h_c = m_i v_i' a, \quad (7)$$

where we have defined a critical impact parameter h_c for which ions have a grazing collision with the object which is

$$h_c = a \left(1 + \frac{2e|\varphi_s|}{m_i v_i^2} \right)^{1/2}. \quad (8)$$

The cross section for the collection of ions is, therefore,

$$\sigma_i = \pi h_c^2 = \pi a^2 \left(1 + \frac{2e|\varphi_s|}{m_i v_i^2} \right) = 4\pi r_o^2 \left(1 + \frac{2e|\varphi_s|}{m_i v_i^2} \right). \quad (9)$$

For a given Maxwellian distribution in the bulk plasma, the ion flux to the object surface is determined by the integration of the corresponding cross-section with $f_i(v_i)$:

$$I_i = n_{io} \int_0^{\infty} v \sigma_i f_i(v) dv. \quad (10)$$

Hence, the ion current I_i^{+} to the grain reads

$$I_i^{+} = Z_i e \left(\frac{m_i}{2\pi K_B T_i} \right)^{3/2} \int_0^{\infty} v \sigma_i e^{-\frac{m_i v^2}{2K_B T_i}} dv. \quad (11)$$

For spherical coordinates, the integration in Eq. (11) performed with the use of the Eq. (10) gives

$$I_i^{+} = 4\pi r_o^2 Z_i n_{oi} e \left(\frac{K_B T_i}{2\pi m_i} \right)^{1/2} \left(1 + \frac{e|\varphi_s|}{K_B T_i} \right). \quad (12)$$

Similarly, for the outward current I_i^{-} we have

$$I_i^{-} = 4\pi r_o^2 Z_i n_{oi} e \left(\frac{m_i}{2\pi K_B T_i} \right)^{3/2} \int_{-\infty}^0 v_z e^{-\frac{m_i v^2}{2K_B T_i}} dv. \quad (13)$$

Hence, we get

$$I_i^- = -4\pi r_o^2 Z_i e n_{oi} \left(\frac{K_B T_i}{2\pi m_i} \right)^{1/2}. \quad (14)$$

Finally, the total ion current reads

$$I_i = I_i^+ + I_i^- = 4\pi r_o^2 Z_i e n_{oi} \left(\frac{K_B T_i}{2\pi m_i} \right)^{1/2} \frac{e|\varphi_s|}{K_B T_i}. \quad (15)$$

Now we can determine the floating potential of the object using the results from above Eq. (15) and equating to the electron current Eq. (5).

$$\left(\frac{T_e}{m_e} \right)^{1/2} \left[1 - \exp \left(-\frac{e|\varphi_s|}{K_B T_e} \right) \right] = \left(\frac{T_i}{m_i} \right)^{1/2} \left(\frac{e|\varphi_s|}{K_B T_i} \right). \quad (16)$$

This new equation (16) is also valid for $|\varphi_s| = 0$. It also represents that when $\beta = T_i/T_e$ increases the normalized potential $\Phi = \frac{e|\varphi_s|}{K_B T_e}$ will also increase as can be seen in Fig. (2). However, one question arises about the equality $n_{oe} \sim Z_i n_{oi}$, since near the surface, due to the collection of ions, the plasma may be inhomogeneous, or $Z_i n_{oi} > n_{oe}$. In addition to this, ions can also lose energy in rare collisions with atoms and become trapped in finite orbits by the electric field of a charged particle[26–28]. Trapped ions effect was studied using molecular dynamics calculations [28] and with the help of analytical methods.[29, 30]. In these papers, it was shown that the density of the trapped ions (with negative total energy) could be greater than the density of the free ions (with positive total energy) in the vicinity of a charged dust particle and thus played an important role in the screening of the particle. How many ions can also be trapped near the object in collisionless plasmas and what is their density distribution is still an unsolved question[21], however in future we intend to include the trapped ions effect in modified theory for the accuracy of model.

Acknowledgments

The results presented here are obtained in the framework of GOA project 2009-009 (K.U.Leuven), the European Commissions Seventh Framework Program (FP7/2007- 2013) under the grant agreement no. 218816 (SOTERIA project, www.soteria-space.eu), Georgian Science Foundation Grant Project No. 1-4/16(GNSF/ST09 305 4-140 and Higher Education Commission of Pakistan.

Figure captions:

Fig. (1): Ions and electrons current to and from the spherical object.

Fig. (2): Normalized potential $\Phi = e|\varphi_s|/K_B T_e$ versus $\beta = T_i/T_e$.

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